Robust Traffic Engineering: Game Theoretic Perspective

Vladimir Marbukh
National Institute of Standards and Technology
100 Bureau Drive, Stop 8920, Gaithersburg,
MD 20899-8920, USA
Tel: 1 301 975 2235
E-mail: marbukh@nist.gov

ABSTRACT
On-line routing algorithms deal with requests as they arrive without assuming any knowledge of the underlying process that generates the streams of requests. By contrast, off-line traffic engineering algorithms assume complete statistical knowledge of the request generating process. This dichotomy, however, oversimplifies many practical situations when some incomplete information on the expected demands is available, and proper utilization of the available information may improve the network performance. This paper proposes a game theoretic framework for robust traffic engineering intended to guard against the worst case scenario with respect to possible uncertainties in the external demands and link loads. The proposed framework can be interpreted as a game of the routing algorithm attempting to optimize the network performance and the adversarial environment attempting to obstruct these efforts by selecting the worst case scenario with respect to the uncertainties. Two different classes of schemes are considered: first, suitable for *MPLS* implementation, centralized schemes, and, second, suitable for *OSPF-OMP* implementation, decentralized schemes.

Categories and Subject Descriptors
C.4 [**Performance of Systems**]: – *modeling techniques, performance attributes, reliability, availability, and serviceability.*

General Terms
Algorithms, Management, Performance, Design, Theory.

Keywords
Uncertain demand, traffic engineering, robustness, game theoretic framework, equal cost multi-path, stability, *MPLS*, *OSPF-OMP*.

1. INTRODUCTION
Consider a network whose performance is characterized by the following penalty function:

$$F = \sum_l f_l(\lambda_l) \tag{1}$$

where the total flow carried on a link l is

$$\lambda_l = \sum_{r:l\in r} x_r \le c_l,\ \forall l, \tag{2}$$

the flow carried on a route r is x_r, the capacity of a link l is c_l, and function $F_l(\lambda_l)$ characterizes penalty associated with carrying load λ_l on link l. Traffic flows $x = (x_r)$ satisfy the following conservation conditions:

$$\sum_{r\in R_{ij}} x_r = \mu_{ij},\ x_r \ge 0,\ \forall r \in R_{ij},\ \forall (i,j) \tag{3}$$

where the set of feasible routes with origin-destination (i,j) is R_{ij}, and the matrix of external demands is $\mu = (\mu_{ij})$. Given, μ, the optimal vector of traffic flows $x = x^*$ minimizes the total penalty (1):

$$\min_x F \tag{4}$$

subject to constraints (1)-(3) [1]. We assume that functions f_l are monotonously increasing and convex:

$$d_l(\lambda) = df_l(\lambda)/d\lambda > 0, \tag{5}$$

$$d_l'(\lambda) = d^2 f_l(\lambda)/d\lambda^2 > 0, \tag{6}$$

$\forall \lambda \in [0,\infty)$, and problem (1)-(4) has at least one feasible solution. These assumptions imply that the optimization problem (1)-(4) has unique optimal solution x^*, no other locally optimal solution exists, and solution x^* can be characterized in terms of the link costs (5) as follows [1]. A set of path flows is optimal if and only if the flows are positive on feasible paths of minimum cost, where the cost of a path is a sum of the costs of the links comprising this path:

$$d_r = \sum_{l\in r} d_l(\lambda_l) \tag{7}$$

This characterization implies that at the optimum, the paths along which the input flow is split must have equal costs and equal to the minimum cost of all feasible paths with the same origin-destination (equal cost multi-path).

Characterization of the optimal routing in terms of the link costs suggests assigning link weights $w = (w_l)$ in the Open Shortest Path First (*OSPF*) routing protocol as follows:

$$w_l = d_l(\lambda_l) \tag{8}$$

Link weight assignments (8) can be used for adaptive *OSPF* implementation, with link loads λ_l estimated from the real-time measurements. If the demand matrix $\mu = (\mu_{ij})$ is fixed and known, off-line implementation of *OSPF* can be based on the pre computed "optimal" link weights $w_l^* = d_l(\lambda_l^*)$, where the average link l load is $\lambda_l^* = \sum_{r:l\in r} x_r^*$, and the optimal traffic assignment x^* is determined by the solution to the optimization problem (1)-(4).

In many practical situations available information on the demand matrix $\mu = (\mu_{ij})$ can be more reliably quantified in term of the "confidence region" $\mu \in M$ rather than point estimate $\mu \approx \tilde{\mu}$. Following [2] we approximately assume that

$$M = \left\{ \sum_{s=1}^{S} \gamma_s \mu^s \middle| \sum_{s=1}^{S} \gamma_s = 1, \gamma_s \geq 0 \right\}. \quad (9)$$

We will refer to μ^s as scenarios, and interpret polyhedron (9) as a mixture of these scenarios with weights $\gamma = (\gamma_s)$.

This paper proposes a game theoretic framework for robust traffic engineering intended to guard against the worst-case scenario with respect to possible uncertainties. Characterization of the external demands μ in terms of the region $\mu \in M$ may be a source of such uncertainty. For an adaptive minimum cost routing unavoidable small variations in the link costs may, in a situation of equal cost multi-path, cause significant variations in the load allocation, and thus contribute to uncertainty in the link loads. The proposed framework can be interpreted as a game of the routing algorithm attempting to optimize the network performance and the adversarial environment attempting to obstruct these efforts by selecting the worst case scenario conditions. Two classes of schemes are considered: first, suitable for *MPLS* implementation, centralized schemes, and, second, suitable for Optimized Multi-Path *OSPF* (*OSPF-OMP*) implementation, decentralized schemes.

2. CENTRALIZED SCHEMES

Broadly speaking, there are two possible frameworks for decision making under uncertainty. The Bayesian framework [3] is concerned with the "average" performance by assuming that uncertain parameters follow some probability distribution. Proposed in [2] approach to *OSPF* link costs (weights) $w = w^*$ optimization under uncertain demands (9) by minimization of the aggregate penalty $F_\Sigma = \Sigma F^s$, where penalty F^s corresponds to scenario $\mu = \mu^s$, $s = 1,..,S$, lies within the Bayesian framework, since criterion F_Σ is, in effect, a result of averaging of the penalty (1) with respect to a random matrix μ over probability distribution $\Pr(\mu = \mu^s) = 1/S$. This approach, however, may not be adequate if one is concerned with the worst rather than average case scenario performance.

Robustness concerns can be addressed within game theoretic framework [4] by identifying the routing pattern that minimizes the worst-case scenario losses in performance, i.e., regrets, due to the uncertainties. A routing protocol is not capable of controlling the flows $x = (x_r)$ under uncertain external demands $\mu \in M$, but hopefully capable of controlling the fractions of the offered load μ to be carries on feasible routes $\xi = (\xi_r)$, where

$$\xi_r = x_r / \mu_{ij}, \; r \in R_{ij} \quad (10)$$

Consider penalty (1) as a function of the fractions ξ and the external demands μ: $F = F(\xi|\mu)$. The loss (regret) resulted from optimization of the routing algorithm for scenario $s = j$ while the actual scenario is $s = i$ can be characterized by

$$L_{ij} = F(\xi^j | \mu^i) - F(\xi^i | \mu^i), \quad (11)$$

$i, j = 1,..,S$, where fractions $\xi^s = (\xi_r^s)$, $\xi_r^s = x_r^s / \mu_{ij}$, $r \in R_{ij}$ are optimized for scenario $s = 1,..,S$. Consider a two player, zero sum game of the routing algorithm attempting to minimize loss (11) by selecting $j = 1,..,S$, and adversarial environment attempting to maximize loss (11) by selecting $i = 1,..,S$. Let α_j be the optimal, generally mixed, strategy for the routing algorithm. It is natural to interpret the weighted sum

$$\xi = \sum_j \alpha_j \xi^j \quad (12)$$

as a robust load allocation scheme guarding against the worst case mixture $\sum_j \beta_j \mu^j$ of scenarios $\mu^i, i = 1,..,S$, where β_i is the optimal, generally mixed, strategy for the environment.

Allocation (12) requires ability to arbitrarily split traffic among feasible routes. In practice it can be achieved in *MPLS* network by randomization of the routing decisions at the packet level. However, it is often desirable to allocate a single route for the entire flow. According to the routing optimality principle, the load should be carried on a minimum cost routes. The following game theoretic interpretation G provides natural extension of this optimality principle to a situation of uncertain external demands. Consider a non-cooperative game G of all origin-destination pairs (i, j) and the adversarial environment. Each pair (i, j) attempts to minimize the excessive, relatively to the minimum, cost of the route

$$V_{ij} = \left[\sum_{l\in r} d_l \left(\sum_{r':l'\in r'} x_{r'} \right) - \min_{r''\in R_{ij}} \sum_{l\in r''} d_l \left(\sum_{r':l'\in r'} x_{r'} \right) \right] \mu_{ij} \quad (13)$$

by selecting a feasible strategy $r \in R_{ij}$. The adversarial environment attempts to maximize the aggregate excessive cost

$$U_\Sigma = \sum_{(i,j)} V_{ij} \quad (14)$$

by selecting a feasible strategy $\mu \in M$. Note that a mixed routing strategy in this game can be interpreted as traffic split at the flow as well as packet level. In a typical situation when each flow occupies a small portion of a link capacity, splitting traffic at the flow and packet levels produce similar results.

3. DECENTRALIZED SCHEMES

The game theoretic interpretation of the optimality principle G can serve as a starting point for developing off-line as well as on-line decentralized, robust traffic engineering schemes. Given fractions (10), uncertainty in the expected demands $\mu \in M$ induce uncertainty in the link weights

$$w_l \in [\breve{w}_l, \widehat{w}_l] \tag{15}$$

where

$$\breve{w}_l = \min_{\mu \in M} d_l \left(\sum_{(i,j)} \mu_{ij} \sum_{r:l \in r \subseteq R_{ij}} \xi_r \right), \tag{16}$$

$$\widehat{w}_l = \max_{\mu \in M} d_l \left(\sum_{(i,j)} \mu_{ij} \sum_{r:l \in r \subseteq R_{ij}} \xi_r \right), \tag{17}$$

and function $d_l(\lambda)$ is given by (5). Assume that vectors $\breve{w} = (\breve{w}_l)$ and $\widehat{w} = (\widehat{w}_l)$ are fixed, and consider a problem of selection of the shortest feasible path $r \in R_{ij}$ in a weighted graph with uncertain link weights (15). It is natural to formalize this problem as a game g_{ij} of the routing algorithm attempting to minimize the excessive, relatively to the minimum, route cost

$$H_{ij}(w,r) = \sum_{l \in r} w_l - \min_{r' \in R_{ij}} \sum_{l \in r'} w_l \tag{18}$$

by selecting a feasible strategy $r \in R_{ij}$, and the adversarial environment attempting to maximize cost (18) by selecting a feasible strategy (15).

In a case of off-line routing the target fractions (10) in (16)-(17) are determined off line, for example, using one of the procedures described in the previous section of the paper. In a case of on-line, adaptive routing bounds (10) are based on the real-time measurements. Given bounds $\breve{w} = (\breve{w}_l)$ and $\widehat{w} = (\widehat{w}_l)$, the robust traffic split for origin-destination (i, j) is determined by the optimal, generally mixed, routing strategy in the game g_{ij}. Note that this optimal routing solution is based on two metrics per link $\breve{w}_l$ and $\widehat{w}_l$, and thus can be implemented with *OSPF-OMP* routing protocol [5].

In conclusion, briefly discuss stability of the routing resulted from solutions to the games g_{ij}. If the optimal solution to the load allocation problem (1)-(4) does not split traffic, this optimal solution can be implemented with *OSPF* routing protocol based on the corresponding "optimal" link weights. If, however, the optimal solution splits traffic among feasible routes, a situation of equal cost multi-path occurs due to the route optimality principle. This situation is typical for moderately and heavily loaded networks with multiple feasible routes since the minimum cost routing increases load on the minimum cost route until the admission strategy takes over or a situation of equal cost multi-path occurs. It is usually assumed that *OSPF* splits traffic equally among minimum cost feasible routes. The problem of adaptive *OSPF* implementation is routing instability (route flapping) [6] due to abrupt changes in the load allocation resulted from small changes in the link weights in a situation of equal cost multi-path. From the game theoretic perspective the route flapping instability can be viewed as an attempt of the minimum cost routing algorithm to solve the corresponding game in pure strategies or strategies describing equal split among some routes. The game theoretic framework provides a natural guiding principle for regularization of the otherwise ill posed problem of route cost minimization in a situation of equal cost multi-path [7]. We currently investigate a problem of global stability of the *OSPF-OMP* routing protocol splitting traffic according to the optimal mixed routing strategy $p_r(\breve{w}, \widehat{w})$ in games g_{ij} for a case when information on the current fractions ξ and feasible demands M is available to the routers. In this case the performance of the corresponding *OSPF-OMP* routing protocol can be described by equations (16)-(17) supplemented with

$$\xi_r = p_r(\breve{w}, \widehat{w}) \tag{19}$$

Other directions of future research include relation between centralized and decentralized game schemes, as well as developing computationally feasible algorithms for solving corresponding games. Solutions for some particular cases have been obtained in [8]-[9].

4. ACKNOWLEDGMENTS

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